Asymptotic behaviors of IDS for random Schrödinger operators associated with Gibbs point processes

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In this talk, we consider the integrated density of states $N(\lambda)$ of the random Schrödinger operators with nonpositive potentials associated with the Gibbs point processes. For some Gibbs point processes, the leading terms of $N(\lambda)$ as $\lambda \downarrow -\infty$ are determined, which are very different from that for a Poisson point process, which is known. This presentation is based on [2].

1 Introduction

We consider the random Schrödinger operator on $L^2(\mathbb{R}^d, dx)$ defined by

$$H_\omega = -\Delta + V_\omega, \quad V_\omega(x) = \sum_{y \in \Gamma(\omega)} u(x-y),$$

where u, called *single site potential*, is a nonpositive continuous measurable function on $\mathbb{R}^d$ with compact support, and Γ is a point process, i.e. a random variable with values in sets of points in $\mathbb{R}^d$.

When Γ is a stationary Poisson point process, the *integrated density of states (IDS)* $N(\lambda)$ of the Schrödinger operator is formally given by

$$\lim_{L \to \infty} \frac{1}{L^d} \#\{\text{eigenvalues of } H^D_{\omega,L} \text{ less than or equal to } \lambda\},$$

where $H^D_{\omega,L}$ is the operator H_ω restricted to the box $(-L/2, L/2)^d \subset \mathbb{R}^d$ with Dirichlet boundary condition. Then the IDS and the spectrum of H_ω are independent of ω almost surely, and $N(\lambda)$ is a nondecreasing function increasing only on the spectrum (see [4]). It is known that

$$\log N(\lambda) \sim \frac{\lambda \log|\lambda|}{|\min u|} \quad (\lambda \downarrow -\infty), \tag{1}$$

which is proved by Pastur (see [3]).

In this talk, we consider the case where Γ is a *Gibbs point process* i.e. point processes with interaction between the points (see [1]), and investigate the asymptotic behaviors of $N(\lambda)$ as $\lambda \downarrow -\infty$.

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2 Main result

If the interaction is sufficiently weak, the asymptotic behavior of the IDS is identical to (1). However, when the interaction is *pairwise interaction*, the behavior can be different.

Theorem 1. *In the case of the pairwise interaction: the energy of the points* $\{x_j\}$ *is*

$$a \sum_{i<j} 1_{[0,R]}(|x_i - x_j|) \quad (a, R > 0),$$

we have

$$\log N(\lambda) \sim -\frac{a}{2\|u\|_R^2}\lambda^2 \quad (\lambda \downarrow -\infty),$$

where

$$\|u\|_R^2 = \sup\{\sum_{j=1}^{\infty} u(x_j)^2 \mid |x_i - x_j| > R \ (i \neq j)\}$$

This implies that the IDS decays much faster than that for a Poisson point process.

References

[1] D. Dereudre, Introduction to the theory of Gibbs point processes. *Stochastic geometry*, 181–229, Lecture Notes in Math., 2237, Springer, Cham, 2019.

[2] Y. Nakagawa, Asymptotic behaviors of the integrated density of states for random Schrödinger operators associated with Gibbs Point Processes, *Preprint*, arXiv:2210.11381 (2022).

[3] L. A. Pastur, The behavior of some Wiener integrals as $t \to \infty$ and the density of states of Schrödinger equations with random potential, *Teor. Mat. Fiz.* **32**, 88–95 (1977) (in Russian).

[4] L. Pastur and A. Figotin, *Spectra of Random and Almost-Periodic Operators*, Grundlehren der mathematischen Wissenschaften, 297, Springer-Verlag, Berlin, 1992