Snapshot problem for the wave equation

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Abstract.

In this talk, we deal with the uniqueness and the existence of the solution to the wave equation $\partial_t^2 u - \Delta u = 0$ on $\mathbb{R}^n$ with several snapshots. More precisely, our problem is formulated as follows. For given times $t_1, \cdots, t_m \in \mathbb{R}$, and for m given smooth functions $f_1, \cdots, f_m$ on $\mathbb{R}^n$, we consider the wave equation

$$\partial_t^2 u(t,x) - \Delta u(t,x) = 0, \qquad (t,x) \in \mathbb{R} \times \mathbb{R}^n,$$

with the condition $u|_{t=t_1} = f_1, \cdots, u|_{t=t_m} = f_m$. It is natural to ask when the above equation has a unique solution. We call the above problem the snapshot problem for the wave equation, and the set of m functions $\{f_1, \cdots, f_m\}$ the snapshot data.

Roughly speaking, our main results are as follows.

(1) If we take two snapshots, namely, $m = 2$, then the uniqueness for the snapshot problem does not hold.

(2) If we take three snapshots, namely, $m = 3$, and if the ratio $(t_3 - t_1)/\pi(t_2 - t_1)$ is irrational, then the uniqueness for the snapshot problem holds.

(3) We assume that $m = 3$ and $(t_3 - t_1)/\pi(t_2 - t_1)$ is irrational and not a Liouville number. In addition, we assume a certain compatibility condition on the snapshot data $\{f_1, f_2, f_3\}$. Then the snapshot problem for the wave equation has a unique solution.

If we have enough time, we would like to mention a generalization to the wave equation on symmetric spaces.

This is a joint work with Jens Christensen, Fulton Gonzalez, and Jue Wang.