Inverse Scattering on the Metric Graph for Graphene

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We consider an inverse scattering problem on the metric graph associated with the hexagonal lattice. Metric graph consists of differential operators on each edge with suitable boundary conditions at every vertex. Here, we consider one-dimensional Schrödinger operators on the edges. We show that the potential can be determined by the scattering amplitude. In order to do this, we introduce an artificial boundary value problem and prove that the potential can be reconstructed from the Dirichlet-to-Neumann map for this boundary value problem.

Let $\Gamma = (\mathcal{V}, \mathcal{E})$ be the hexagonal lattice where $\mathcal{V}$ is the vertex set and $\mathcal{E}$ is the edge set. Every edge $\boldsymbol{e} \in \mathcal{E}$ is identified with the interval $[0, 1]$. Roughly speaking, the Schrödinger operator on $\mathcal{E}$ is a family of differential operators $-d^2/dx^2 + q_{\boldsymbol{e}}(x)$ defined on $\boldsymbol{e}$, where $q_{\boldsymbol{e}}(x) = \overline{q_{\boldsymbol{e}}(x)} \in L^2(0, 1)$. We also assume that $q_{\boldsymbol{e}}(x)$, i.e. $q_{\boldsymbol{e}}(x) = q_{\boldsymbol{e}}(1 - x)$, for every edge $\boldsymbol{e}$, and $q_{\boldsymbol{e}} = 0$ except for a finite number of edges. Moreover, we impose the Kirchhoff condition on every vertices.

We can define the scattering amplitude $A(\lambda)$ associated with the Schrödinger operator of the metric graph for $\lambda \in (0, \infty) \setminus \mathcal{T}$. Here $\mathcal{T}$ is some discrete set. The scattering amplitude $A(\lambda)$ is defined passing through the singularity expansion of generalized eigenfunctions in the momentum space. This method is used in the study of discrete Schrödinger operators on the periodic lattices.

The main result is the following:

Theorem 1. *Suppose that we know the scattering amplitude $A(\lambda)$ associated the Schrödinger operator for $\lambda \in I$ where I is an open interval in $(0, \infty) \setminus \mathcal{T}$. Then we can determine $q = \{q_{\boldsymbol{e}}\}_{\boldsymbol{e} \in \mathcal{E}}$ uniquely.*

References

[1] Kazunori Ando, Hiroshi Isozaki, Evgeny Korotyaev, and Hisashi Morioka. Inverse Scattering on the Metric Graph for Graphene. preprint